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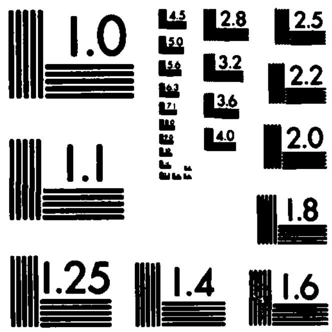

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During the period covered by this report the investigators continued their research on the development and application of numerical methods for singularly-perturbed (or stiff) boundary value problems for ordinary differential equations and initial-boundary value problems for partial differential equations. Results were obtained for collocation methods for vector systems of two-point boundary value problems and for adaptive grid finite element methods to several interesting physical problems, such as, the deformation of nonlinear elastic and plastic beams and a nonlinear Schrodinger equation which exhibits self focusing.

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AFOSR-TR- 83-0810

INTERIM SCIENTIFIC REPORT

Air Force Office of Scientific Research Grant AFOSR-80-0192

Period: 1 June 1982 through 31 May 1983

Title of Research: Numerical Methods for Singularly Perturbed Differential Equations with Applications

Principal Investigator: Joseph E. Flaherty

Department of Mathematical Sciences
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Troy, New York 12181

ABSTRACT

During the period covered by this report we continued our research on the development and application of numerical methods for singularly-perturbed (or stiff) boundary value problems for ordinary differential equations and initial-boundary value problems for partial differential equations. Results were obtained for collocation methods for vector systems of two-point boundary value problems and for adaptive grid finite element methods for partial differential equations. We are applying our methods to several interesting physical problems, such as, the deformation of nonlinear elastic and plastic beams and a nonlinear Schrodinger equation which exhibits self focusing.

1. Progress and Status of the Research on Numerical Methods for Singularly Perturbed Differential Equations

During the period covered by this report we continued our research on the development and application of numerical methods for singularly-perturbed ordinary and partial differential equations. Our progress and publications in these areas are described below.

1.1 Boundary Value Problems for Ordinary Differential Equations

Flaherty and O'Malley continued their collaboration on using asymptotic analysis to develop numerical methods for singularly perturbed two-point boundary value problems. Two papers [2, 3][1] describing their findings on nonlinear vector systems with endpoint boundary layers have appeared, a third paper [4] will appear in the proceedings of the Tenth IMACS World Congress, and a fourth [5] has been submitted to the SIAM Journal on Scientific and Statistical Computing. They have applied their methods to some difficult nonlinear stiff examples and have found them to be both accurate and efficient. Their approach seems to be particularly well suited to problems involving the deformation of nonlinear elastic beams. It has been the basis of further numerical work by others, including Professors U. Ascher of the University of British Columbia and R. Weiss of the Technical University of Vienna.

We recently began studying the possibility of extending our methods to two-point boundary value problems with turning points. In particular, William Harte, a graduate student under the direction of Flaherty and O'Malley,

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[1] See the list of publications and abstracts at the end of this report.

recently completed an MS (Computer Science) project on "Numerical Solutions of Perturbed Boundary Value Problems". He developed an adaptive finite difference code that uses the algebraic transformations of O'Malley[2] to separate the fast and slow components of the differential system so that each part can be discretized in a stable manner. His program also adaptively concentrates the computational mesh in regions where the truncation error is determined to be large.

Harte's code is not yet applicable to turning point problems, although we feel that it is an important first step in this direction. The main problem is that different discretization techniques must be used for the decaying and growing fast components of the system. G. M. Heitker, a graduate student under Flaherty and O'Malley's direction, has been studying the possibility of using splines under tension and special collocation and finite difference schemes to overcome this difficulty. Similar schemes were found to be successful on scalar problems with turning points (cf. Flaherty and Mathon)[3].

1.2. Initial-Boundary Problems for Partial Differential Equations.

Flaherty and several graduate students have been continuing their research on adaptive finite element methods for partial differential equations. Their findings on one-dimensional moving grid schemes for parabolic problems will appear in the Proceedings of the ARO Workshop on Adaptive Methods for Partial Differential Equations (cf. [6]).

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[2] R. E. O'Malley, Jr., "Slow/fast decoupling for linear boundary value problems," Lecture Notes in Math. 985 (1983), 248-266.

[3] J. E. Flaherty and W. Mathon, "Collocation with polynomial and tension splines for singularly perturbed two-point boundary value problems", SIAM J. Sci. Stat. Comput., Vol. 1 (1980), pp. 260-289.

J. M. Coyle, one of the authors of this paper, is a graduate student who is being supported by this grant. A paper [1] on using splines under tension to construct explicit finite difference and finite element schemes for hyperbolic partial differential equations appeared in the International Journal for Numerical Methods in Engineering. We have also been conducting a weekly seminar on adaptive techniques for differential equations.

Reference [6] contains several applications, including a focusing problem for the nonlinear Schrodinger equation. This problem describes the self-focusing of a laser beam in a medium with a nonlinear index of refraction and it is being done in collaboration with Professor A. C. Newell of the University of Arizona. It is a difficult numerical problem because the amplitude of the solution becomes infinite as focusing occurs. Our adaptive finite element code appears to be able to cope with this difficulty and is giving very good results. We will discuss the physical and additional computational aspects of this problem in a forthcoming paper by Coyle, Flaherty, and Newell [7].

We are studying several improvements and additions to our adaptive finite element code. For one-dimensional problems, J. M. Coyle and R. Ludwig have been examining mesh adaptation and equidistribution strategies, J. M. Coyle has also been developing a p-version adaptive finite element code, P. Moore is implementing procedures that perform local time and spatial step refinements in regions of high error, and S. Adjerid is working on adaptive procedures for a posteriori error estimation. For two-dimensional problems, Cpt. D. Rochette completed his MS project on "Generalized Conjugate Gradient Residual Methods and Similar Descent Methods for Solving Nonsymmetric Systems of Linear Equations", Maj. D. C. Arney is implementing adaptive mesh moving and equidistribution procedures, R. Ludwig is studying local step refinement procedures, and Cpt. J. Hayes has been implementing a two- and three-dimensional graphics package. Mr. Coyle is being partially supported by this grant; Maj. Arney and Cpt. Hayes are U. S. Army officers who are on educational leave from the U. S. Military Academy; and Cpt. Rochettte completed his MS degree in Applied Mathematics this May and is currently a professor at the U. S. Military Academy.

When completed, this research should produce some of the most powerful codes for solving time dependent partial differential equations.

This grant provides funds for a visitor, and this year Stephen Davis of ICASE, NASA Langley Research Center, visited R.P.I. for one week. He gave a lecture on adaptive methods and conducted research with Flaherty, Coyle, and Ludwig, which resulted in a paper (cf. [6]).

2. Interactions.

Professor Flaherty presented lectures on material pertaining to this grant at the following conferences and organizations:

Tenth IMACS World Congress, Concordia University, Montreal, 8-13 August 1982: lectured on "The numerical solution of boundary value problems for stiff differential equations".

ARO Workshop on Adaptive Methods for Partial Differential Equations, University of Maryland, 14-16 February 1983: lectured on "Adaptive finite element methods for parabolic partial differential equations". Maj. D. C. Arney, Mr. J. M. Coyle, and Mr. R. Ludwig attended this workshop. Flaherty will also be an editor (with Dr. J. Chandra and Professor I. Babuska) of the conference proceedings.

Sandia Laboratories, 15 April 1983: lectured on "Adaptive finite element methods". Also consulted with Dr. R. C. Y. Chin of Lawrence Livermore Laboratories and Professors J. Oliger and J. B. Keller of Stanford University as part of this trip.

3. List of publications and Manuscripts in Preparation

Publications

1. J. E. Flaherty, "A Rational Function Approximation for the Integration Point in Exponentially Weighted Finite Element Methods", Int. J. Num. Meth. Engng., Vol 18 (1982), pp 775-797. Also, Tech. Rep. ARLCB-TR-81022, Benet Weapons Laboratory, Watervliet Arsenal, Watervliet, New York, June 1981.

2. J. E. Flaherty and R. E. O'Malley, Jr., "Singularly Perturbed Boundary Value Problems for Nonlinear Systems, Including a Challenging Problem for a Nonlinear Beam", in W. Eckhaus and E. H. deJager (Eds.), Proc. Conf. on Singular Perturbations and Applics., Lect. Notes in Maths. No. 924, Springer-Verlag, Berlin, 1982, pp. 170-191.

3. J. E. Flaherty and R. E. O'Malley, Jr., "Asymptotic and Numerical Methods for Vector Systems of Singularly-Perturbed Boundary Value Problems, in Proc. 1982 Army Numer. Anal. and Comp. Conf., ARO Report 82-3, pp 381-395, 1982. Also, Tech. Rep. ARLCB-TR-82031, Benet Weapons Laboratory, Watervliet Arsenal, Watervliet, New York, October 1982.

In Press

4. R. E. O'Malley, Jr. and J. E. Flaherty, "On the Numerical Solution of Singularly-Perturbed Boundary Value Problems", to appear in Trans. Tenth IMACS World Conf., Montreal, Que., August 1982.

5. J. E. Flaherty and R. E. O'Malley, Jr., "Numerical Methods for Stiff Systems of Two-Point Boundary Value Problems", NASA Contractor Report 166115, NASA Langley Research Center, Hampton, VA, April 1983. Also, submitted to SIAM J. Sci. and Stat. Comput., April 1983.

6. J. E. Flaherty, J. M. Coyle, R. Ludwig, and S. F. Davis, "Adaptive Finite Element Methods for Parabolic Partial Differential Equations", to appear in I. Babuska, J. Chandra, and J. E. Flaherty (Eds.), Proc. ARO Workshop on Adaptive Methods for Partial Differential Equations, SIAM, Philadelphia, 1983.

In Preparation

7. J. M. Coyle, J. E. Flaherty, and A. C. Newell, "Focusing Problems for Damped and Undamped Nonlinear Schrodinger Equation", in preparation for Physica D.

A RATIONAL FUNCTION APPROXIMATION FOR THE INTEGRATION POINT IN EXPONENTIALLY WEIGHTED FINITE ELEMENT METHODS

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ABSTRACT

A rational function is presented for approximating the function $f(z) = \coth z - 1/z$ that appears in several exponentially fitted or weighted finite difference and finite element methods for convection-diffusion problems. The approximation is less expensive to evaluate that $f(z)$ and provides greater accuracy than the doubly asymptotic approximation when $z = O(1)$.

Int. J. Num. Meth. Engng., Vol. 18 (1982), pp 775-797. Also Tech. Rep. ARLCB-TR-81022, Benet Weapons Laboratory, Watervliet Arsenal, Watervliet, New York, June, 1981.

SINGULARLY PERTURBED BOUNDARY VALUE PROBLEMS FOR NONLINEAR SYSTEMS, INCLUDING A CHALLENGING PROBLEM FOR A NONLINEAR BEAM

by

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ABSTRACT

We consider certain singularly perturbed two-point boundary value problems involving nonlinear vector systems

$$\dot{x} = f(x,y,t,\varepsilon), \quad \varepsilon\dot{y} = g(x,y,t,\varepsilon)$$

of $m + n$ ordinary differential equations on a finite interval $0 \leq t \leq 1$ subject to q initial conditions and r terminal conditions of the form

$$A(x(0),y(0),\varepsilon) = 0, \quad B(x(1),y(1),\varepsilon) = 0,$$

with $q + r = m + n$. Most critically, in addition to natural smoothness assumptions, we assume that the $n \times n$ Jacobian matrix $g_y(x,y,t,0)$ has a hyperbolic splitting with $k > 0$ stable eigenvalues (i.e., eigenvalues having strictly negative real parts) and $n - k > 0$ (strictly) unstable eigenvalues for all x and y and $0 \leq t \leq 1$. We suppose that $q \geq k$ and $r \geq n - k$ and we find limiting solutions as the small positive parameter ε tends to zero.

We apply our asymptotic methods to study the deformation and stresses in a thin nonlinear elastic beam resting on a nonlinear elastic foundation. Results are presented for simple, clamped, and elastic support conditions.

In W. Eckhaus and E. H. de Jager (Eds.), Proc. Conf. on Singular Perturbations and Applic., Lect. Notes in Math., No. 924, Springer-Verlag, Berlin, 1982, pp 170-191.

ASYMPTOTIC AND NUMERICAL METHODS FOR VECTOR SYSTEMS OF SINGULARLY-PERTURBED BOUNDARY VALUE PROBLEMS

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ABSTRACT

Procedures are developed for constructing asymptotic solutions for certain nonlinear singularly-perturbed vector two-point boundary value problems having boundary layers at one or both end points. The asymptotic approximations are generated numerically and can either be used as is or to furnish a two-point boundary value code (e.g. COLSYS) with an initial approximation and a nonuniform computational mesh. The procedures are applied to several examples involving the deformation of nonlinear elastic beams.

In Proc. 1982 Army Numer. Anal. and Comp. Conf., ARO Report 82-3, pp 381-395, 1982. Also Tech. Rep. ARLCB-TR-82031, Benet Weapons Laboratory, Watervliet Arsenal, Watervliet, New York, October 1982.

ON THE NUMERICAL SOLUTION OF SINGULARLY-PERTURBED VECTOR BOUNDARY VALUE PROBLEMS

by

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ABSTRACT

Numerical procedures are developed for constructing asymptotic solutions of certain nonlinear singularly-perturbed vector two-point boundary value problems having boundary layers at one or both end points. The asymptotic approximations are generated numerically and can either be used as is or to furnish a two-point boundary value code (e.g. COLSYS) with an initial approximation and a nonuniform computational mesh. The procedures are applied to a model problem that indicates the possibility of multiple solutions and problems involving the deformation of a thin nonlinear elastic beam resting on a nonlinear elastic foundation.

To appear in Trans. IMACS Conf., Montreal, Que., Aug. 1982.

NUMERICAL METHODS FOR STIFF SYSTEMS OF

TWO-POINT BOUNDARY VALUE PROBLEMS

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ABSTRACT

We develop numerical procedures for constructing asymptotic solutions of certain nonlinear singularly perturbed vector two-point boundary value problems having boundary layers at one or both endpoints. The asymptotic approximations are generated numerically and can either be used as is or to furnish a general purpose two-point boundary value code with an initial approximation and the nonuniform computational mesh needed for such problems. The procedures are applied to a model problem that has multiple solutions and to problems describing the deformation of a thin nonlinear elastic beam that is resting on an elastic foundation.

Submitted to SIAM J. Sci. and Stat. Comput., April 1983. Also NASA Contractor Report 166115, ICASE, NASA Langley Research Center, Hampton, April 1983.

ADAPTIVE FINITE ELEMENT METHODS

FOR PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT

We discuss a finite element method for solving initial-boundary value problems for vector systems of partial differential equations in one space dimension and time. The method automatically adjusts the computational mesh as the solution evolves in time so as to approximately minimize the local discretization error. We are thus able to calculate accurate solutions with fewer elements than would be necessary with a uniform mesh.

Our overall method contains two distinct steps: a solution step and a mesh selection step. We solve the partial differential equations using a finite element-Galerkin method on trapezoidal space-time-elements with either piecewise linear or cubic Hermite polynomial approximations. A variety of mesh selection strategies are discussed and analyzed. Results are presented for several computational examples.

To appear in the Proceedings of the <u>ARO Workshop on Adaptive Methods for Partial Differential Equations</u>, I. Babuska, J. Chandra, and J. E. Flaherty (Eds.), SIAM, Philadelphia, 1983.

FOCUSING PROBLEMS FOR A NONLINEAR SCHRODINGER EQUATION

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ABSTRACT

We consider a cylindrically symmetric Schrodinger equation with a cubic nonlinearity. It is known that this equation has solutions that self-focus the initial data is strong enough. We study this problem numerically using a self-adaptive finite element code and seek to determine (i) the quantitative nature of the solution as it focuses and (ii) whether the solution will still focus in the presence of a small amount of dissipation.

In preparation for Physica D.

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